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# Effective action: massive and massless fields*

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**Abstract**
We present a brief review of some recent results concerning quantum corrections to the action of gravity. The main attention is paid to the massless limit in the vacuum effects of massive matter fields, in particular to the discontinuity phenomenon for the massive Abelian fields and the conformal anomaly in the metric-scalar theory. Furthermore we discuss, for a while in a qualitative way, the (non)universality of loop corrections in effective quantum gravity.

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## 1. Introduction

Quantum gravity is one of the most challenging and difficult problems in the modern theoretical physics. One attractive option is to use the concept of induced gravity, which is to start from the theory where the action of gravity is induced by quantum effects of some other fields or objects. The most radical realization of this concept is the string theory, where all known 'fundamental' fields and interactions between them are induced by the quantum effects of the really fundamental objects: two-dimensional strings living in the multidimensional target space. Obviously, the most difficult part here is to fit with the results of QED, standard model and other successful quantum field theories. It is supposed that all these theories are effective ones. This means that the corresponding classical and quantum effects are independent of the fundamental theory (e.g., superstring) which dominates at the high energy scale.

If all known fundamental interactions, including gravity, are induced ones and if the effective quantum field theory approach is a right thing for the matter fields, does it mean the gravity also must be quantized and must be a subject of effective field theory approach? This is not necessary so. We know that the role of gravity is quite different from that of the matter-field theories and, therefore, it might happen that gravity does not need to be quantized at all. Of course, one cannot be certain about that at the present state of art. On the other hand,

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if gravity must be quantized, it should be an effective quantum theory, in the same fashion as the successful matter-field theories are. So, we have a real choice between two distinct background concepts. But one thing is quite certain here: the relation between massive and massless theories is the issue of primary importance. This relation includes such aspects at the massless limit in the massive quantum theories, violation of scale and conformal symmetries by quantum anomalies and, of course, the decoupling of massive fields at low energies. In fact, those are the issues which form the basis of effective field approach and hence it is absolutely important to understand them as well as possible in the gravitational setting. In this paper, we shall review some recent advances in the understanding of these problems.

## 2. Renormalization in curved spacetime

In order to define the notions which will be used in what follows, let us present a brief description of renormalization in curved spacetime. The introduction to QFT in curved space can be found in many books and reviews, e.g., in [1–3].

The first step is to consistently formulate quantum theory of matter on classical curved background. The qualitatively new element is the action of vacuum, that is of an external gravitational field. The standard criterium for the action of external field are locality of the vacuum action, renormalizability[2] and simplicity, the last means one does not need to introduce more vacuum terms than necessary. For example, this means we can construct the vacuum actions without parameters with the inverse mass dimensions. In this way we arrive at the vacuum action

$$S_{\text{vac}} = S_{\text{EH}} + S_{\text{HD}}, \tag{1}$$

where

$$S_{\text{EH}} = -\frac{1}{16\pi G}\int \mathrm{d}^4x\,\sqrt{g}\,\{R + 2\Lambda\} \tag{2}$$

is the usual gravity action including the Einstein–Hilbert term and the nonzero cosmological constant. The theory based on this action passed almost all known experimental and observational tests and is commonly seen as the basis of classical relativistic gravitational physics. The second term in (1) contains higher derivatives,

$$S_{\text{HD}} = \int \mathrm{d}^4x\,\sqrt{-g}\{a_1C^2 + a_2E + a_3\Box R + a_4R^2\}, \tag{3}$$

where $C^2 = R^2_{\mu\nu\alpha\beta} - 2R^2_{\alpha\beta} + (1/3)R^2$ is the square of the Weyl tensor and $E = R^2_{\mu\nu\alpha\beta} - 4R^2_{\alpha\beta} + R^2$ is the integrand of the Gauss–Bonnet topological term. $a_{1,2,3,4}$ are dimensionless parameters of the vacuum action. Let us remark that the presence of higher derivative terms is unavoidable in a renormalizable theory in curved space. The same concerns the cosmological term, especially in the case when matter fields are massive. The reason is that the higher derivative terms and the cosmological constant do arise as divergences due to quantum corrections. Therefore, if one does not introduce them from the very beginning into the classical action, these divergences cannot be removed by the usual renormalization procedure. On the top of the mentioned metric-dependent terms, in order to provide renormalizability in the interacting theory with scalar field, $\varphi$, one has to introduce the nonminimal term, $\xi R\varphi^2$, where $\xi$ is the nonminimal parameter (subject of renormalization). For multiple scalars, $\varphi_k$, the corresponding term looks like $\xi_{ik}R\varphi^i\varphi^k$.

[2] Here we request the multiplicative renormalizability of the theory, which enables one, in principle, to perform calculation of the physical observables in a controllable way. The covariance of the counterterms is guaranteed by the general theorems constructed for the gauge theories in [4, 5].

The reason why the higher derivative terms in (3) were not observed yet in the gravitational experiments is that the lower derivative Einstein–Hilbert term has coefficient $1/16\pi G = M_P^2$. In order to understand the role of this coefficient, one can use the Feynman diagrams. For example, the Newton law is the consequence of one-graviton exchange between two masses. Then, in order to produce a relevant impact on the Newton law, the additional degrees of freedom associated with higher derivatives (see, e.g., [3]), the energy of graviton should be comparable to the Planck energy.

## 3. Quantum corrections in the massive case and decoupling for the interacting scalar theory

The divergences can be removed by the multiplicative renormalization of the matter fields, couplings, masses, nonminimal parameters $\xi_{ik}$ and the parameters of the vacuum action, $\Lambda, G, a_{1,2,3,4}$. The next step is to calculate the relevant finite part of the quantum corrections. This problem, in general, is very far from being solved, despite the serious efforts in this direction were made starting from early 1970s [6]. In the special case of free massless conformal fields, the vacuum effective action (EA) can be obtained by integrating the conformal anomaly [7]. The anomaly-induced EA [8] is an exact result for the particular FRW metric, where it leads to the Starobinsky inflationary model [9] or to its alternative versions based on the effective field theory approach [10]. The complete form of induced action [11] also enables one to classify the vacuum states of the black hole [12] in a regular way. The reason why the massless conformal case is so special is that for the massless fields there is a certain duality between UV and IR regimes. The consequence of this property is that the essential part of the most important nonlocal sector of the EA is controlled by the local part, corresponding to divergences. Basically, the counterterms define the trace anomaly and the most relevant (by assumption, except the FRW metric) part of the EA can be obtained by integrating the anomaly. One can find the detailed technical review of anomaly and anomaly-induced effective action in [13] and hence we will not repeat it here.

The situation with the quantum corrections for the massive fields is essentially more complicated. In fact, there are no regular methods of deriving the nonlocal sector of the EA, except the one based on the renormalization group [3, 14]. However, the renormalization group based method is presumably valid only in the UV regime while for the EA in the IR and intermediate regimes this method is not reliable. Let us remark that it is very important to know the EA out of the UV regime. The investigation of this issue could support or disprove the possibility of the running cosmological constant [15] and also help to discriminate between the cosmological models based on different sorts of such running [16–18]. Furthermore, the insufficient information about the contributions of massive fields to the vacuum EA do not enable further development of the modified Starobinsky model of inflation [10], which is potentially the most natural inflationary model.

The purpose of this brief review is to present some part of the recent achievements of authors and our collaborators in understanding the mentioned quantum corrections [19–21]. The one-loop Euclidean effective action is given by the formula (see, e.g., the book [3])

$$\bar{\Gamma}^{(1)} = -\tfrac{1}{2}\,\mathrm{Tr}\ln\hat{H},$$

where $\hat{H}$ is the bilinear form of the matter-fields action. The calculation of EA can be performed using Feynman diagrams in the framework of linearized gravity [19] or it can be based in the heat kernel method [23]. Let us consider, as a first example, the result of the calculation of the EA in the second order in generalized curvatures for the interactive massive scalar field with the nonminimal coupling [20],

$$\bar{\Gamma}^{(1)}_{\rm scalar} = \frac{1}{2(4\pi)^2}\int d^4x\, g^{1/2}\left\{\frac{m^4}{2}\cdot\left(\frac{1}{\epsilon}+\frac{3}{2}\right)+\left(\xi-\frac{1}{6}\right)m^2R\left(\frac{1}{\epsilon}+1\right)\right.$$
$$+\frac{1}{2}C_{\mu\nu\alpha\beta}\left[\frac{1}{60\epsilon}+k_W(a)\right]C^{\mu\nu\alpha\beta}+R\left[\frac{1}{2\epsilon}\left(\xi-\frac{1}{6}\right)^2+k_R(a)\right]R$$
$$\left.-\frac{\lambda}{2\epsilon}m^2\phi^2+\phi^2\left[\frac{\lambda^2}{8\epsilon}+k_\lambda(a)\right]\phi^2+\phi^2\left[-\frac{\lambda}{2\epsilon}\left(\xi-\frac{1}{6}\right)+k_\xi(a)\right]R\right\}, \qquad (4)$$

where the parameter $\epsilon$ of dimensional regularization is

$$\frac{1}{\epsilon}=\frac{1}{2-\omega}+\ln\left(\frac{4\pi\mu^2}{m^2}\right)-\gamma$$

and we used notations [19]

$$A = 1-\frac{1}{a}\ln\frac{1+a/2}{1-a/2}, \qquad a^2=\frac{4\,\Box}{\Box-4m^2}. \qquad (5)$$

The formfactors corresponding to the parameters $\lambda$ and $\xi$ have the form

$$k_\lambda(a)=\frac{A\lambda^2}{4} \qquad \text{and} \qquad k_\xi(a)=\lambda\left[\frac{A(a^2-4)}{12a^2}-\frac{1}{36}-A\left(\xi-\frac{1}{6}\right)\right].$$

These two formfactors contain all information about the scale dependence of the parameters $\lambda$ and $\xi$ and, in general, represent the finite nonlocal part of the corresponding sectors of the EA. The UV limit corresponds to $a\to 2$ and the IR limit to $a\to 0$. It is easy to check that in the IR limit $A\sim -a^2/12\to 0$ and therefore both $k_\lambda(a)$ and $k_\xi(a)$ tend to zero quadratically in $a$. This is a direct manifestation of the decoupling theorem [24] which holds also in curved space. One can observe it also on the expressions for the formfactors $k_W(a)$ and $k_R(a)$ in [19]. The gravitational version of the Appelquist and Carazzone theorem holds also for massive fermions and vectors, including the case of spontaneous symmetry breaking [25].

The physical $\beta$-functions for the parameters $\lambda$ and $\xi$ demonstrate some deviation from those based on the minimal subtraction scheme of renormalization. For example, in the physical scheme, the Landau pole related to $\lambda$-coupling is shifted in comparison to the MS-scheme (see, e.g., [20]).

## 4. Discontinuity in the quantum corrections for the Proca field

The next interesting issue which we shall treat here is the massless limit for the quantum corrections derived in the framework of massive quantum theory. In many cases the limit $m\to 0$ is smooth in both divergences and the nonlocal formfactors. But there are some special cases when one can meet the discontinuity in the quantum corrections. These situations emerge when the mass of the field is forbidden by some symmetry. Two most important examples of such particles are photon and graviton. In this section, we shall mainly follow the original paper [21] and consider in details the case of photon and then present the qualitative discussion of the difference with the case of graviton.

At the classical level, the situation with the photon is as follows. The gauge invariance rules out the existence of the photon mass, but this is indeed a theoretical restriction only. What about experiment? The upper bound for the photon mass is impressively low: $m < 10^{-25}$ GeV for the estimates based on the soft breaking of the gauge symmetry and $m < 10^{-23}$ GeV for the Higgs mechanism based origin of the photon mass. One can think that this strong limit is sufficient to close the problem, but there is a subtle point. The number of physical degrees of freedom of the massive photon is larger than that for the massless photon. Now, since an extra degree of freedom is actually almost massless, it may provide an additional contribution to the interactions mediated by the massive photon[3].

We can look at the problem from the other side and compare the quantum contributions of massless and massive photons to the vacuum EA. Consider the massive Abelian vector fields, which is also called the Proca model. The action of the theory in curved space has the form

$$S_P = \int d^4x\sqrt{-g}\left\{-\frac{1}{4}F_{\mu\nu}^2 + \frac{1}{2}M^2A_\mu^2\right\}. \tag{6}$$

On the way to deriving the quantum corrections one has to deal with the softly broken gauge invariance. The bilinear operator in quantum fields,

$$\hat{H} = H_\alpha^\mu = \delta_\alpha^\mu\Box - \nabla_\alpha\nabla^\mu - R_\alpha^\mu - M^2\delta_\alpha^\mu, \tag{7}$$

is degenerate while the theory is not invariant under the standard gauge transformation. The noninvariance does not permit the use of the usual Faddeev–Popov technique. The known way of solving this problem [26] requires introducing an auxiliary operator,

$$\hat{H}^* = H^{*\nu}_{\ \ \mu} = -\Box\delta_\mu^\nu + M^2\delta_\mu^\nu, \tag{8}$$

which satisfies the following two properties:

$$H_\alpha^\mu H^{*\nu}_{\ \ \mu} = M^2\left(\delta_\alpha^\nu\Box - R_\alpha^\nu - M^2\delta_\alpha^\nu\right),$$
$$\mathrm{Tr}\ln\hat{H}^* = \mathrm{Tr}\ln(\Box - M^2).$$

As a result we arrive at the following relation:

$$-\mathrm{Tr}\ln\hat{H} = -\mathrm{Tr}\ln\left(\delta_\alpha^\nu\Box - R_\alpha^\nu - M^2\delta_\alpha^\nu\right) + \mathrm{Tr}\ln(\Box - M^2). \tag{9}$$

An obvious advantage of the last formula is that both operators at the rhs are not degenerate and admit a simple use of the standard Schwinger–DeWitt technique for the divergences and even the use of a more advanced method for deriving the nonlocal terms in the second order in curvature approximation [19, 23].

One can observe certain similarity between the expression (9) and the known formula for the massless case. In both cases we meet contributions from minimal vector and scalar operators. Indeed, the second contribution in (9) is analogous to the ghost contribution in the massless case, but there is a factor 1 instead of a factor $1/2$ in front of the term $\mathrm{Tr}\ln\Box$ in the strictly massless case. As a result of this difference one can observe a discontinuity in the vacuum contribution of massive vector field in the massless limit. The difference between the $M \to 0$ limit in equation (9) and the contribution of a massless vector is exactly equal to the contribution of a minimal massless scalar.

In order to understand the origin of this scalar better, let us consider a new action

$$S_P' = \int d^4x\sqrt{-g}\left\{-\frac{1}{4}F_{\mu\nu}^2 - \frac{1}{2}M^2\left(A_\mu - \frac{1}{M}\partial_\mu\varphi\right)^2\right\}. \tag{10}$$

The remarkable property of this expression is the gauge invariance under the transformations

$$A_\mu \to A_\mu' = A_\mu + \partial_\mu\xi \qquad \text{and} \qquad \varphi \to \varphi' = \varphi + \xi M.$$

Furthermore, in the special gauge $\varphi = 0$ we come back to the Proca field action (6). And finally, since both (6) and (10) are free fields actions, the gauge-fixing dependence is irrelevant for the quantum correction which depends only on the external metric field.

[3] Let us remark that there is an alternative way to introduce massive photon, related to the fermion loop corrections or to string theory which can, presumably, lead to the violation of Lorentz and/or CPT symmetries [22].

The last observation means that it is not necessary to perform practical calculations in the $\varphi = 0$ gauge. Instead, one can choose another gauge, e.g., the one which simplifies the Feynman diagrams or the Schwinger–DeWitt technique. Let us use the linear gauge-fixing condition $\chi = \nabla_\mu A^\mu - M\varphi$ for deriving the quantum corrections. Then the sum of the action (10) and the FP gauge fixing term, $S_{gf} = -\frac{1}{2}\int d^4x\sqrt{-g}\,\chi^2$, is cast into the factorized form

$$S' = \int d^4x\sqrt{-g}\,\left\{A^\alpha\left(\delta^\nu_\alpha\Box - R^\mu_\alpha - M^2\delta^\mu_\alpha\right)A_\nu + \varphi(\Box - M^2)\varphi\right\}.$$

A simple calculation of the gauge ghost operator gives

$$\hat{H}_{\text{gh}} = \Box - M^2$$

and therefore the one-loop effective action is given by

$$\begin{aligned}\bar{\Gamma}^{(1)} &= -\tfrac{1}{2}\,\mathrm{Tr}\ln\left(\delta^\nu_\alpha\Box - R^\mu_\alpha - M^2\delta^\mu_\alpha\right)\\ &\quad - \tfrac{1}{2}\,\mathrm{Tr}\ln(\Box - M^2) + \mathrm{Tr}\ln(\Box - M^2),\end{aligned} \tag{11}$$

that is nothing but (9). One can see that an extra scalar was indeed 'hidden' in the massive term of the vector. At this point, we conclude that the Stückelberg procedure [27] described above works also in curved spacetime and represents a useful alternative to the scheme (8)–(9) for the Proca field in curved spacetime [26].

Now we are in a position to discuss the discontinuity in the quantum contributions to the vacuum effective action from the Proca model in the massless limit. For this end, we remember the second order in curvature result for the Proca field derived in [19],

$$\begin{aligned}\bar{\Gamma}^{(1)}_{\text{vector}} &= \frac{1}{2(4\pi)^2}\int d^4x\, g^{1/2}\left\{\frac{3}{2}M^4\cdot\left(\frac{1}{\epsilon}+\frac{3}{2}\right) + \frac{M^2}{2}R\left(\frac{1}{\epsilon}+1\right)\right.\\ &\quad\left. + \frac{1}{2}C_{\mu\nu\alpha\beta}\left[\frac{13}{60\epsilon} + k^v_W(a)\right]C^{\mu\nu\alpha\beta} + R\left[\frac{1}{72\epsilon} + k^v_R(a)\right]R\right\}.\end{aligned} \tag{12}$$

The nonlocal finite part of the effective action is characterized by the two formfactors

$$\begin{aligned}k^v_W(a) &= -\frac{91}{450} + \frac{2}{15a^2} - \frac{8A}{3a^2} + A + \frac{8A}{5a^4},\\ k^v_R(a) &= -\frac{1}{2160} + \frac{A}{48} + \frac{A}{3a^4} + \frac{1}{36a^2} - \frac{A}{18a^2}.\end{aligned} \tag{13}$$

The massless limit in the formfactor $k^v_R(a)$ provides an interesting information about conformal anomaly [13, 28] and on the corresponding ambiguity. However, the most interesting from the point of view of physical interpretation is another formfactor, $k^v_W(a)$. This term defines the leading-log quantum contribution to the propagation of the gravitational wave, that is the transverse traceless part of the gravitational perturbation $h_{\mu\nu} = g_{\mu\nu} - \eta_{\mu\nu}$, where $\eta_{\mu\nu}$ is the flat background metric. In the limit $M \to 0$ the coefficient inside the Weyl-square term becomes

$$\frac{13}{60\epsilon} + k^v_W(a) \quad\to\quad \frac{13}{60}\left(\frac{1}{2-w} - \gamma - \ln\frac{\Box}{4\pi\mu^2}\right) - \frac{38}{225}. \tag{14}$$

The divergent term is cancelled by counterterm and the finite constant terms may be included into the renormalization of the $C^2_{\mu\nu\alpha\beta}$ term in the classical vacuum action. Finally, the most relevant term is of course the nonlocal one $-\frac{13}{60}\ln\frac{\Box}{4\pi\mu^2}$. This term is a physical contribution to the gravitational wave equation for the massless limit of the Proca model. On the other hand, the corresponding term derived for the gauge vector field is just $-\frac{1}{5}\ln\frac{\Box}{4\pi\mu^2}$. The difference between the two coefficients is $1/60 = 13/60 - 1/5$ is nothing else but the contribution of an extra scalar field which we have discussed before. In the massless limit this field does not disappear and gives contribution to the vacuum effective action. This effect demonstrates the discontinuity in the massless limit for the quantum contributions of the massive (Proca) vector field.

One can compare the discontinuities in massive spin-1 and massive spin-2 cases [21]. Free massive $s \geqslant 2$ field models are interesting examples of the softly broken gauge theories. It is well known that the discontinuity takes place already at the classical level [29]; however, this effect disappears when the consideration is performed in curved space, e.g., on de Sitter.

In this situation the discontinuity at the quantum level is especially important. Evaluation of EA for the spin-2 field has been performed via the Stückelberg procedure similar to that we used for the spin-1 case [30]. The divergent parts of the vacuum EA are really different for the massless case and for the massless limit of the massive field. Is it sufficient to claim discontinuity?

Let us remember that the Lagrangian for the free massive $s = 2$ field is known only for the spaces of constant curvature. But in such spaces nonlocal insertions do not exist. Furthermore, any difference in the coefficient of the local term in the EA can be fixed by the renormalization condition. Then, the difference between the theories with different coefficients of the local terms disappears.

Finally, although the difference between the divergent contributions in the spin-2 case can be seen as an indication to the discontinuity, it does not automatically imply that this is really a physical phenomenon.

## 5. Effective approach and universality of quantum gravity

In this section, we shall present some qualitative considerations concerning the effective quantum field theory approach to gravity. It is well known that the construction of the consistent perturbative quantum gravity models meets serious obstacles. In short, the quantum gravity based on general relativity is not renormalizable. The same property is shared, for instance, by all $f(R)$ models. On the other side, there are renormalizable models of quantum gravity, but they have higher derivatives in all sectors of the propagator of the gravitational perturbation and, most important, this includes the gauge-fixing independent spin-2 sector of the theory. The most popular higher derivative quantum gravity model is based on the action

$$S = -\int d^4x\sqrt{g}\left\{\frac{1}{2\lambda}C^2 - \frac{1}{\rho}E + \frac{1}{\xi}R^2 + \tau\,\Box R - \frac{1}{\kappa^2}(R + 2\Lambda)\right\}, \tag{15}$$

where $\kappa^2 = 16\pi G$, $\Lambda$ is the cosmological constant and $\lambda$, $\rho$, $\xi$, $\tau$ are independent parameters in the higher derivative sector of the action. This theory is renormalizable [31] and asymptotically free [32–36], but it is not unitary at the tree level due to the emergence of the massive pole in the propagator of the spin-2 excitations [31],

$$G(k) = \frac{1}{k^2\left(k^2 + m_2^2\right)} = \frac{1}{m_2^2}\left(\frac{1}{k^2} - \frac{1}{k^2 + m_2^2}\right). \tag{16}$$

The negative sign in the last term indicates the presence of a particle with negative kinetic energy. An attempt to exclude this particle from the spectrum leads to the nonunitary $S$-matrix. Let us remark that including more higher derivatives terms into the action may lead to the super-renormalizable theory of quantum gravity, but the unphysical poles remain in the spectrum anyway [37]. There is a chance that the quantum corrections may change the situation [33, 38, 39] and this possibility has not been ever disproved, but this is not either certain [40].

Before we start to discuss the universality of quantum gravity theories, let us make two comments on the problem of unitarity and unphysical ghosts. The first point concerns the relation between renormalizability and unitarity. Let us remember that the complete theory of quantum gravity should involve also the quantum effects of the matter fields. As we already learned in section 2, the renormalizable theory of quantum matter in curved space should start from the classical action which includes the vacuum term (1). It is easy to recognize that this action is nothing else but (15), therefore the higher derivative terms should be included into the action in any case, except in the theory where all quantum effects are derived directly from string theory. The necessity to quantize matter fields in the framework of effective field theory approach makes the special parametrization in the string-induced gravitational action [41–43]. The second point is the natural question: is it true that the usual condition of the unitarity of the *S*-matrix is a valid criterium for quantum gravity? The theory of higher derivative quantum gravity with zero cosmological constant is not renormalizable and, therefore, the action of the theory has to include the cosmological constant. Let us remember that the flat space is not a classical solution for the gravity theory based on the action (15) in the case when the cosmological constant term is nonzero. Then, the possibility to formulate *in* and *out* states for the quantum metric and hence the applicability of the usual *S*-matrix approach is not obvious. An alternative formulations of quantum field theory, like those proposed in [44], may be an appropriate instrument here.

One can look at the problem of renormalizability in quantum gravity from another viewpoint. It is well known that there are quantum field theory methods, which can be called effective approach, which enable one to extract some relevant information from the nonrenormalizable theories (see, e.g., the book [45]. The main idea is that the quantum effects of heavy degrees of freedom become irrelevant at low energies due to decoupling (similar to that we have discussed in section 3).

Recently, there were some interesting papers concerning the possibility to treat quantum gravity in the effective framework [46]. The first observation is that the Newton law may be reproduced by the nonrelativistic limit of the single Feynman diagram representing a one-graviton exchange between two masses. The second step is to add the loop corrections, which provide the quantum correction to the Newton force and maybe other observables. It turns out that the IR effects of quantum gravity can be perfectly well separated from the UV ones and, therefore, from the UV divergences. As a result one ends up with the well-defined quantum corrections to the desirable observable, e.g., to the Newton potential.

One of the most important aspects of effective approach to quantum gravity is the statement of its universality. According to [46] whatever form the fundamental theory of quantum gravity takes, the low-energy correction to the Newton potential is defined by the quantum general relativity. If this is really so, this would mean real progress in our understanding of quantum gravity. At the same time, there are some reasons to suspect that this statement may not be correct. Let us consider the case of higher derivative theory (15). The universality of IR quantum gravity, in this context, is equivalent to the universality of the massless mode which provides the Newton law at the tree level. However, in the case of higher derivative theory (15), this property holds only at the classical level, where it is simply reduced to the uniqueness of the massless part of the propagator of the spin-2 mode of the metric. At the same time, there is no obvious reason to assume universality beyond the tree level. For instance, one has to take into account the interaction vertices between the massless spin-2 modes, and these vertices are not the same for the general relativity and the higher derivative theory (15). The argument about covariance does not look convincing, especially because the vertices are not covariant objects. Another way to express the same idea is to say that the covariance holds for the whole action (15) but not necessary for the low-energy sector inside the loops. Definitely, this issue is not trivial, but represents a problem which deserves a special careful treatment. The only way to check the universality of quantum gravity is to perform an explicit calculation, as it was suggested in [36]. We hope to come back to this problem soon.

## 6. Conclusions

We have considered some aspects of the interface between massive and massless quantum field theories in curved spacetime. The problem of deriving quantum corrections to the vacuum EA from the massive fields is very important and opens the door for numerous applications in cosmology and also the perspective of better understanding of quantum gravity and its possible (non)universality. In conclusion, this area is very promising and the number of unsolved interesting and important problems is enormous.

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